関数解析的学習ユニット　　　ハクアンミン
Functional Analytic Learning Unit
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Geometry of Covariance Matrices and Covariance Operators

Goal: Unified formulations for the different distances and divergences between symmetric positive definite (SPD) matrices and operators

Recent work: H.Q.M. Infinite-dimensional Log-Determinant divergences III: Log-Euclidean and Log-Hilbert-Schmidt divergences. **Information Geometry and Its Applications IV, 2018**

Alpha-Beta Log-Euclidean divergences between positive definite matrices

$$D_{\odot}^{(\alpha,\beta)}(A,B) = \frac{1}{\alpha\beta}\log\det\left[\frac{\alpha(A\odot B^{-1})^{\beta}+\beta(A\odot B^{-1})^{-\alpha}}{\alpha+\beta}\right], \quad \alpha>0,\beta>0$$

Limiting case: Log-Euclidean distance

$$\lim_{\alpha\to 0} D_{\odot}^{(\alpha,\alpha)}(A, B) = \frac{1}{2}||\log(A)-\log(B)||_F^2$$

Alpha-Beta Log-Hilbert-Schmidt divergences between positive definite operators on an infinite-dimensional Hilbert space

$$D_{r,\odot}^{(\alpha,\beta)}[(A+\gamma I),(B+\mu I)] = \frac{1}{\alpha\beta}\log\left[\left(\frac{\gamma}{\mu}\right)^{r(\delta-\frac{\alpha}{\alpha+\beta})}\det{}_X\left(\frac{\alpha(Z+\frac{\gamma}{\mu}I)^{r(1-\delta)}+\beta(Z+\frac{\gamma}{\mu}I)^{-r\delta}}{\alpha+\beta}\right)\right]$$

where $Z+\frac{\gamma}{\mu}I = (A+\gamma I)\odot(B+\mu I)^{-1}$ and $\delta = \frac{\alpha\gamma^r}{\alpha\gamma^r+\beta\mu^r}$.

Limiting case: Log-Hilbert-Schmidt distance

$$\lim_{\alpha\to 0} D_{2\alpha,\odot}^{(\alpha,\alpha)}[(A+\gamma I),(B+\mu I)] = \frac{1}{2}||\log(A+\gamma I)-\log(B+\mu I)||_{\mathrm{HS}_X}^2$$

Under review

1) H.Q.M. Regularized divergences between covariance operators and Gaussian measures on Hilbert spaces, 2018

2) H.Q.M. Infinite-dimensional Log-Determinant divergences between positive definite Hilbert-Schmidt operators, 2018

3) H.Q.M. A unified formulation for the Bures-Wasserstein and Log-Euclidean/Log-Hilbert-Schmidt distances between positive definite operators, 2019