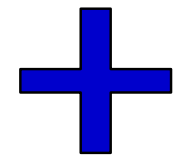


## What is causal discovery?

- Methodology for **inferring causal graphs** using data

### Assumptions

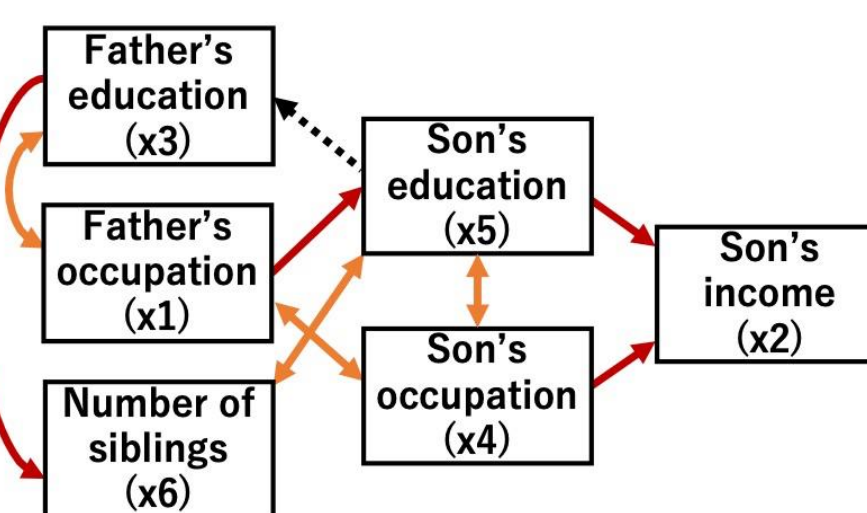
- Functional form?
- Distribution?
- Hidden common cause present?
- Acyclic? etc.



### Data

| x1   | x2     | x3 | x4   | x5 | x6 |
|------|--------|----|------|----|----|
| 87.9 | 45433  | 17 | 76.3 | 17 | 1  |
| 87.9 | 55071  | 16 | 86   | 18 | 2  |
| 62.1 | 113159 | 16 | 87.9 | 16 | 0  |
| 78.5 | 30289  | 16 | 30.1 | 14 | 4  |
| 32.3 | 113159 | 20 | 63.5 | 20 | 7  |
| 60.6 | 55071  | 17 | 83.7 | 17 | 1  |
| 76.4 | 55071  | 16 | 78   | 14 | 2  |
| 63.5 | 37173  | 12 | 63.2 | 16 | 3  |
| 63.2 | 113159 | 14 | 86.5 | 17 | 1  |
| 36.5 | 37173  | 12 | 83.7 | 12 | 4  |

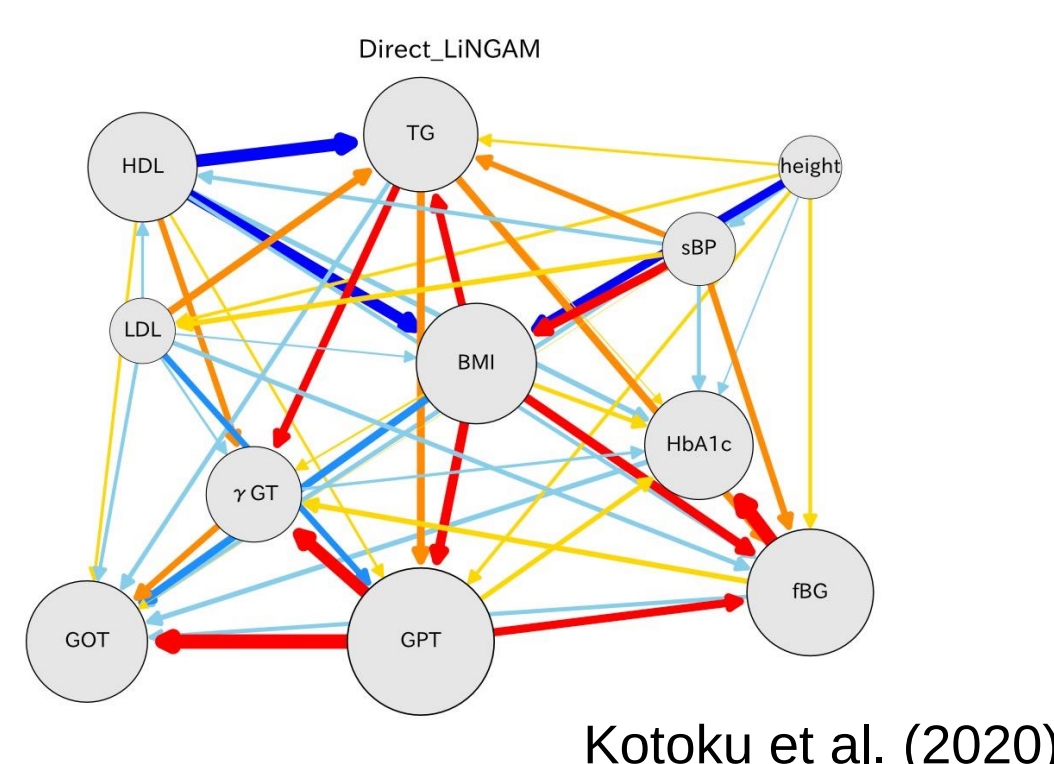
### Causal graph



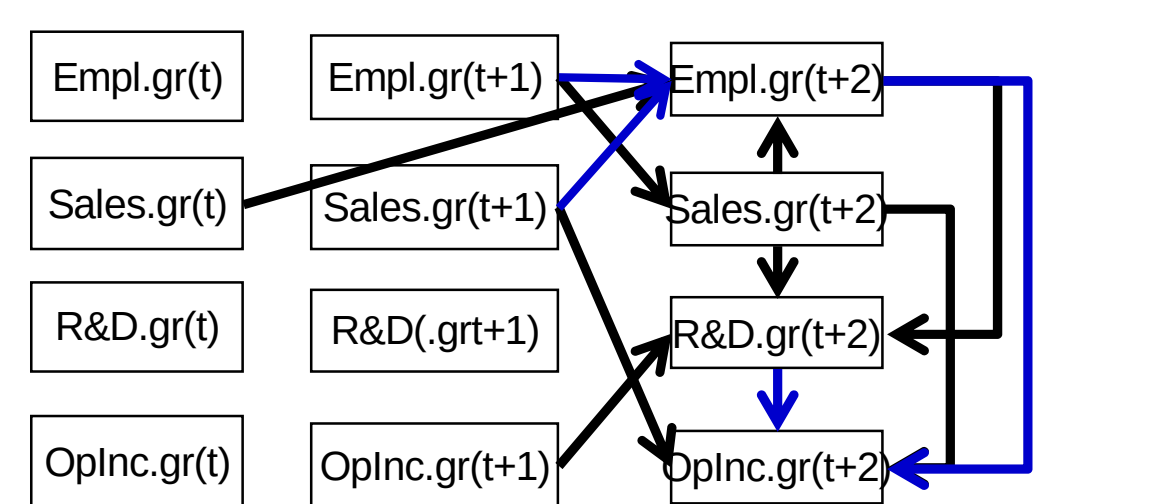
Maeda and Shimizu (2020)

## Application examples

- Biology (Maathuis et al., 2010)
- Prevention medicine (Kotoku et al., 2020)
- Chemistry (Campomanes et al., 2014)
- Material Science (Nelson et al., 2021)
- Climatology (Liu et al., 2020)
- Economics (Moneta et al., 2013)
- Psychology (von Eye et al., 2012)
- Policymaking (高山ら, 2021)
- Network log data (Jarry et al., 2021)



Kotoku et al. (2020)



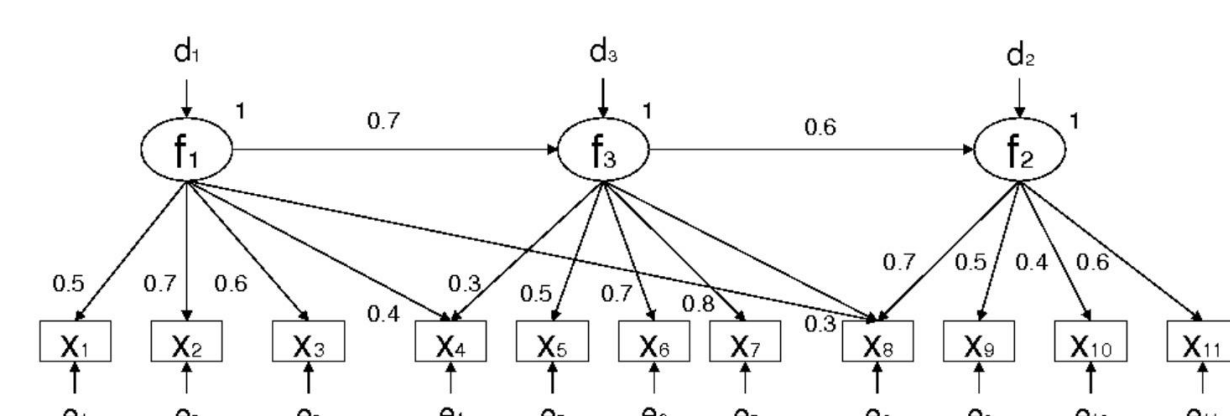
Moneta et al. (2013)

<https://www.shimizulab.org/lingam/lingampapers/applications-and-tailor-made-methods>

## LiNGAM for latent factors (Shimizu et al., 2009)

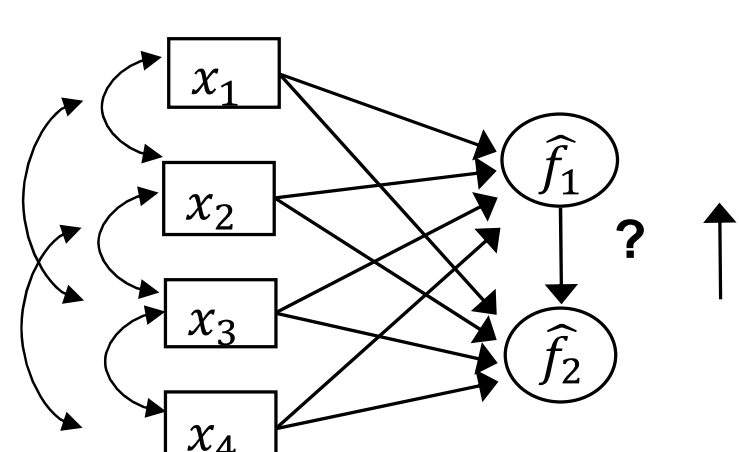
- Model:

$$\begin{aligned} f &= Bf + \epsilon \\ x &= Gf + e \end{aligned}$$



- 2 pure measurement variables per latent needed to identify the measurement model (Silva et al., 2006; Xie et al., 2020)

- Estimate the latent factors and then their causal graph



## Find common and unique factors across multiple datasets (Zeng et al., 2021)

- Model

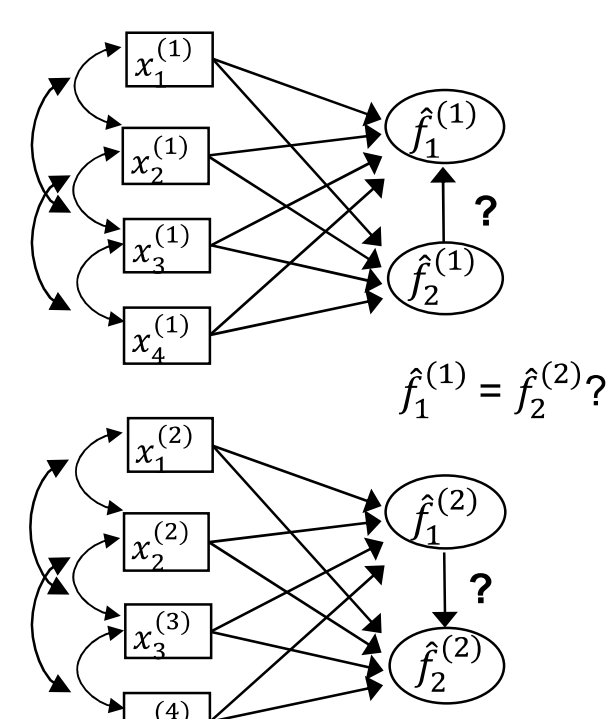
$$\begin{aligned} f^{(m)} &= B^{(m)} f^{(m)} + \epsilon^{(m)} \\ x^{(m)} &= G^{(m)} f^{(m)} + e^{(m)} \end{aligned}$$

$m = 1, \dots, M$

- Score function: likelihood + DAGness (Zheng et al., 2018)

DAGness (Zheng et al., NeurIPS2018):  $h(B) = \text{tr}(e^{B \circ B}) - N$  factors

- Feature extraction across multiple datasets + causal discovery of latent factors



Y. Zeng, S. Shimizu, R. Cai, F. Xie, M. Yamamoto, Z. Hao. **Causal discovery with multi-domain LiNGAM for latent factors**. In *Proc. the 30th International Joint Conference on Artificial Intelligence (IJCAI2021)*, 2021.

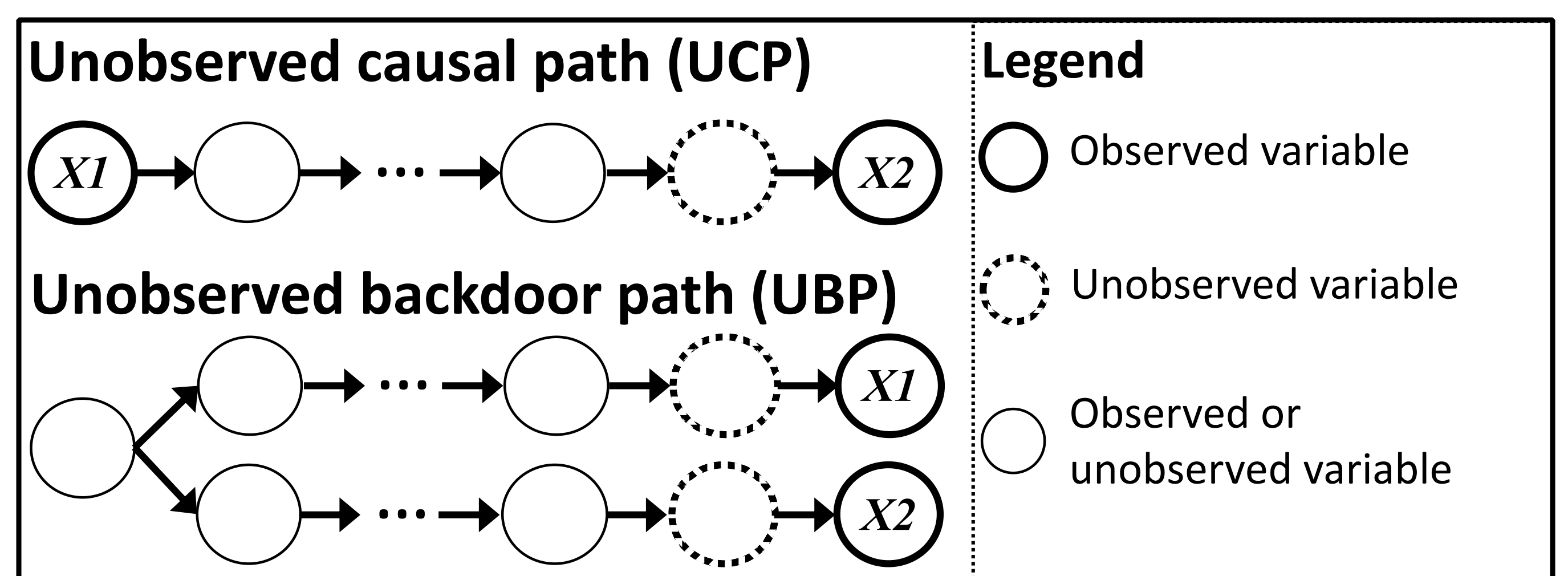
## Causal additive models with unobserved variables

### Identifiability of indirect causal relationships in linear and nonlinear cases

- Assume that X1 has an indirect causal effect on X3 mediated through unobserved variable Y2.
- In linear cases:
  - The residual obtained by the regression of X3 on X1 is **independent** of X1.
- In nonlinear cases (Cascade additive noise models):
  - The residual obtained by the regression of X3 on X1 **cannot be independent** of X1.



### Cases in which causal relationships between variables cannot be identified



## Causal additive models with unobserved variables (CAM-UV)

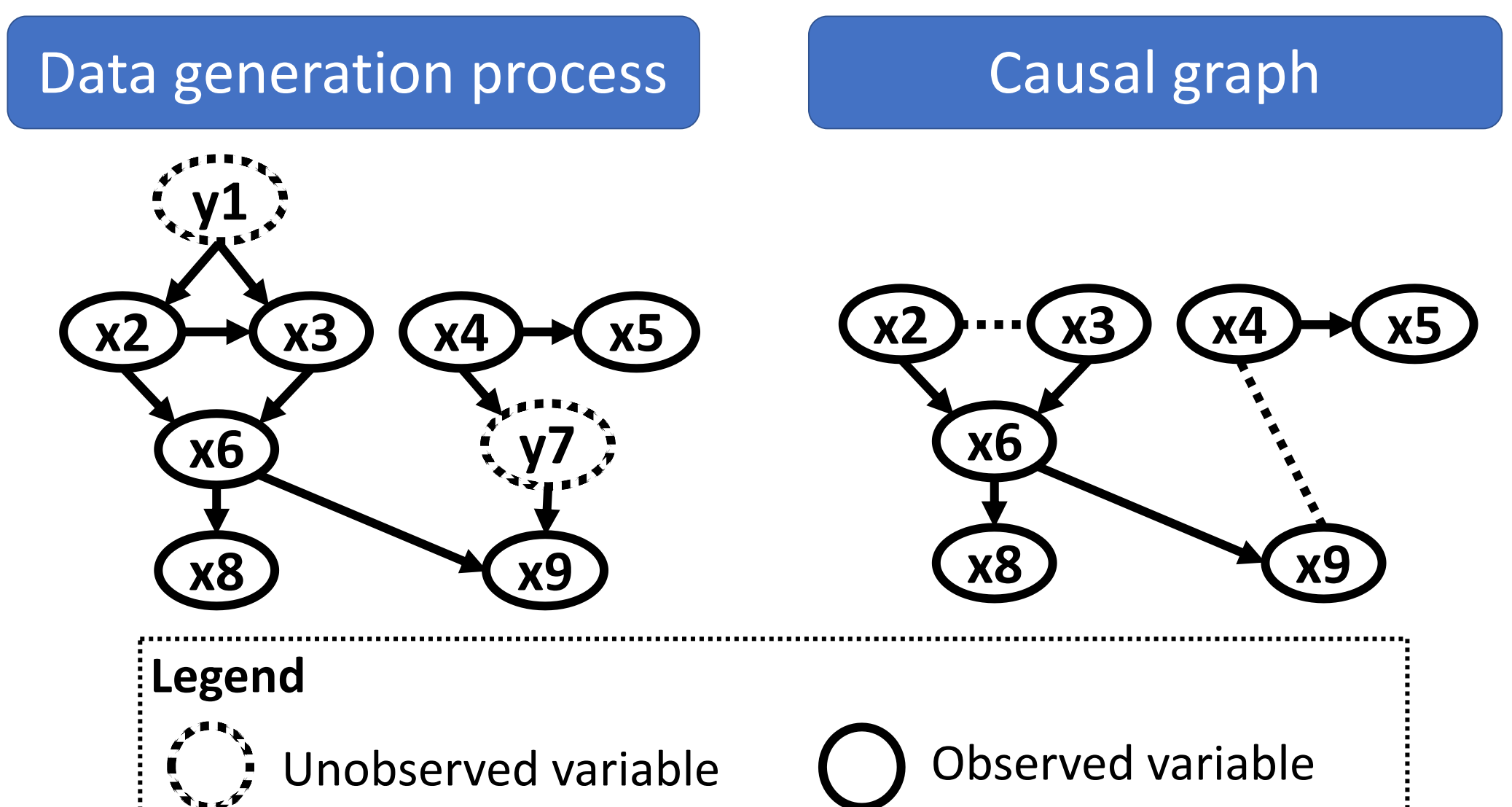
$$v_i = \sum_{x_j \in P_i} f_j^{(i)}(x_j) + \sum_{y_k \in Q_i} f_k^{(i)}(y_k) + n_i$$

- $v_i$ : Observed or unobserved variable
- $x_i$ : Observed variable
- $y_i$ : Unobserved variable
- $n_i$ : External effects
- $f$ : Nonlinear function
- $P_i$ : Direct observed cause of  $x_i$
- $Q_i$ : Direct unobserved cause of  $x_i$ .

The effects of unobserved variables

## The objective of CAM-UV

- Infer the causal directions of all the variable pairs that do not have a UCP or UBP.



T. N. Maeda, S. Shimizu. **Causal additive models with unobserved variables**. In *Proc. 37th Conf. on Uncertainty in Artificial Intelligence (UAI2021)*, 2021